Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

THE MATHEMATICS TEACHER

VOLUME XV JANUARY 1922 NUMBER 1

THE HUMAN WORTH OF RIGOROUS THINKING*

By Professor CASSIUS J. KEYSER
Columbia University

The question I purpose to discuss briefly is this: how much mathematical discipline is essential to the appropriate education of men and women as *human* beings?

This very important question admits of a definite answer and it admits of it in terms of a supremely important and incontestable general principle. Before stating the principle I will give it a name. I shall call it the Principle of Humanistic Education as Distinguished from what is called Industrial or Vocational Education. I say "as distinguished from" because the two varieties of education differ widely in the conceptions at the heart of them and in the motives that actuate them.

In order to set the principle of humanistic education in clear light, let me indicate briefly the obvious facts lying at its base and leading naturally to its formulation. What the individuals composing our race have in common falls into two parts: a part consisting of those numerous instincts, impulses, propensities and powers that we humans have in common, not only with one another, but with many of the creatures of the animal world—a *sub*-human world**; and a second part consisting of such instincts, impulses, propensities and powers as are distinctively human. These latter constitute our Common Humanity. They present an endless variety of detail but in the long course of man's experience with man he has learned to group them, in accordance with their principal aspects, into a small number of familiar classes. And so the nature of our common humanity is fairly well characterized by saying that human beings as such possess in some recognizable measure such marks as the following: a sense for language, for expression in speech—the literary faculty; a sense for the past, for the value of experience—the historical faculty; a sense for the future, for natural law, for prediction—the scientific faculty; a sense for fellowship, co-operation, and justice—the political faculty; a sense for the beautiful—the artistic faculty; a sense for logic, for rigorous thinking—the mathematical faculty; a sense for wisdom, for world harmony, for cosmic understanding—the philosophical faculty; and a sense for the mystery of divinity—the religious faculty. Such are the evident tokens and the cardinal constituents of that which in human beings is human. It is essential to note that to each of these senses or faculties there corresponds a certain type of distinctively human activity—a kind of activity in which all human beings, whatever their stations or occupations, are obliged to participate. Like the faculties to which they correspond, these types of activity, though they are interrelated, are yet distinct. Each of them has a character of its own. Above each of the types of activity there hovers an *ideal* of excellence—a guardian angel wooing our loyalty with a benignant influence superior to every compulsive force and every authority that may command. Nothing more precious can enter a human life than a vision of those angels, and it is the revealing of them that humanistic education has for its function and its aim. Stated in abstract terms the principle is this: Each of the great types of distinctively human activity owns an appropriate standard of excellence; it is the aim and function of humanistic education to lead the pupil into a clear knowledge of these standards and to give him a vivid and abiding sense of their authority in the conduct of life.

---

* Short address by Professor C. J. Keyser at the meeting, December 2, of the New York Section of the Association of Teachers of Mathematics of the Middle Atlantic States and Maryland.

** It is the duty of every teacher and parent to read and ponder Korzybski's "Manhood of Humanity" (E. P. Dutton & Co.), where it is shown that humans are not a species of animal.

It is plain that this conception stands in sharp contrast with the central idea of vocational or industrial education. For humanistic education has for its aim, as I have said, the attainment of excellence in the great matters that constitute our common humanity. On the other hand, industrial education is directly and primarily concerned with our individualities, and so it might more properly be called individualistic education. It regards the world as an immense camp of industries where endlessly diversified occupations call for special propensities, gifts and training. And so its aim, its ideal, is to detect in each youth as early as may be the presence of such gifts and propensities as tend to indicate and to qualify him for some specific form of calling or bread-winning craft; then to counsel and guide him in the direction thereof; and finally, by way of education, to teach him those things which, in the honorable sense of the phrase, constitute "the tricks of the trade."

What are we to say of it? The answer is obvious. Industrial or vocational education, rightly conceived, is essentially compatible with the humanistic type; it may breathe the humanistic spirit; the two varieties of education are essential to constitute an ideal whole, for human beings possess both individuality and the common humanity of man. Industrial education, when thus regarded as supplementary to humanistic education, is highly commendable; but when it is viewed as an equivalent for the latter or as a good-enough substitute for it, it is ridiculous, vicious and contemptible. For the fact must not be concealed that a species of education which, in producing the craftsman, neglects the man, is, in point of kind and principle, precisely on a level with that sort of training which teaches the monkey and the bear to ride a bicycle or the seal to balance a staff upon its nose or to twirl a disc.

It is plain that *one* of the great types of distinctively human activity—perhaps the greatest of all the types—is what is known as Thinking. It consists in the handling of ideas as ideas—the forming of concepts, the combining of concepts into higher and higher ones, discerning the relations among concepts, embodying these relations in the forms of judgments or propositions, ordering these propositions in the construction of doctrine regarding life and the world. It is essential to the argument I am submitting to keep steadily in mind that this kind of activity, our sense for it, our faculty for it, the needs to which it ministers, the joy it gives, and the obligation it imposes are part and parcel of what we have been calling our common humanity. Thinking is not indeed essential to *life*, but it is essential to ***human*** life. All men and women as human beings are inhabitants of the *Gedankenwelt*—they are native citizens, so to speak, of the world of ideas, the world of thought. They must think in order to be human.

And now what shall we say is the ideal of excellence that hovers above thought-activity? What is the angel that woos our loyalty to what is best in that? What is the muse of life in the great art of thinking? An austere goddess, high, pure, serene, cold towards human frailty, demanding perfect precision of ideas, perfect clarity of expression, and perfect allegiance to the eternal laws of thought. In mathematics the name of the muse is familiar: it is Rigor—Logical Rigor—which signifies a kind of silent music, the still harmony of ideas, the intellect's dream of logical perfection.

Can the dream be realized? I am well aware that most of the things which constitute the subject matter of our human thinking—that most of the things to which our thought is drawn by interest or driven by the exigencies of life—are naturally so vague, so indeterminate, that they cannot be handled in strict accord with the rigorous demands of logic. I am aware that these demands cannot be *fully* satisfied even in mathematics, the logical science *par excellence.* Nevertheless I hold that, as the ideal of excellence in thinking, logical rigor is supremely important, not only in mathematical thinking, but in all thinking and *especially* in just those subjects where precision is least attainable. Why? Because without that ideal, thinking is without a just standard for self-criticism; it is without light upon its course; it is a wanderer like a vessel at sea without compass or star. Were it necessary, how easy it would unfortunately be to cite endless examples of such thinking from the multitudinous writings of our time. Indeed, if the pretentious books produced in these troubled years by men without logical insight or a sense of logical obligation were gathered into a heap and burned, they would thus produce, in the form of a bright bonfire, the only light they are qualified to give.

Now it so happens that the term mathematics is the name of that discipline which, because it attains more nearly than any other to the level of logical rigor, is better qualified than any other to reveal the prototype of what is best in the quality of thinking as thinking. And so, in accord with the principle of humanistic education, we have to say that the amount of mathematical discipline essential to the appropriate education of men and women as human beings, is the amount necessary to give them a fair understanding of Rigor as the standard of logical rectitude and therewith, if it may be, the spirit of loyalty to the ideal of excellence in the quality of thought as thought.

Such is, in brief, my answer to the question with which I began. It is, you observe, a qualitative answer in terms of a great human ideal and a sovereign principle of education. If I must add a word touching the strictly quantitative aspect of the question, if I must, that is, attempt to estimate the extent of courses and the length of time necessary and sufficient to yield the required quality and degree of training, I do so with less confidence and far less interest. For so much, so very much, depends on the pupil's talent and the quality of the instruction. A considerable degree of native mathematical talent is much more common than is commonly supposed. Born mathematical imbeciles are not numerous. I venture to say, regarding time and extent of courses, that, for pupils of fair talent, a collegiate freshman year or even a high school senior year of geometry and algebra, if the subjects be administered in the true mathematical spirit, with due regard to precision of ideas and to the exquisite beauty of perfect demonstration, is sufficient to give a fair vision of the ideal and standard of sound thinking.*

---

* Readers will be interested to know that the foregoing remarks are but a fragment of the opening chapter of Professor Keyser's forthcoming book, "Mathematical Philosophy" (E. P. Dutton & Co.)—Ed.